MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

EEM 2036 – ENGINEERING MATHEMATICS III

(All sections / Groups)

02 JUNE 2017
9:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 8 pages with 4 Questions only.
2. Attempt all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given .
3. Please write all your answers in the Answer Booklet provided

QUESTION 1

(a) Solve the linear system of three equations using Gordan Jordan Elimination

$$6x + 2y - 2z = 2$$

$$2x + 2y + 4z = 12$$

$$4x + 4y + 6z = 20$$

[18marks]

(b) Let $A = \begin{pmatrix} 7 & 3 & 2 \\ 0 & 5 & 1 \\ 2 & 7 & 1 \end{pmatrix}$ and $p(t) = 5t^2 + 7t - 8$

Solve the matrix problem. Find p(A). [7marks]

Continued ...

QUESTION 2

(a) Evaluate $\int_1^4 e^{2x} + \frac{2}{x+4}\, dx$ by using Composite Simpsons rule with 3 intervals. Determine the error bound. [15marks]

(b) Given the nodes $x_0 = -2, x_1 = 5, x_2 = 7, x_3 = 9$, find the Lagrange interpolating polynomial for

$$f(x) = \frac{7}{1+x^2}$$

Hence use the Lagrange interpolating polynomial to find the interpolation value for $x = 6$.

[10marks]

Continued ...

QUESTION 3

(a) Find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 25$ and below by the plane $z = 4$ by using cylindrical coordinates.

[10marks]

(b) Evaluate the integral

$$\iint_R y - 2x^2 dA$$

where R is the region bounded by the square $|x| + |y| = 2$

[15marks]

Continued …

QUESTION 4

(a) Evaluate $\int_C (2x^2 - y)dx + xy^3 dy$ where C consists of the segment from (0,1) to (4,1), (4,1) to (4,-1) and (4,-1) to (0,-1).

[10marks]

(b) Use Gauss Theorem to evaluate

$$\vec{F} = xi + xy^2 j - xyzk$$

where the region is inside the solid cylinder $x^2 + y^2 = 9$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$

[15marks]

Continued ...

APPENDIX

TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0)\cdots(x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued ...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}(f(a)+f(b)) - \frac{h^3 f''(\xi)}{12}$$

for some ξ in (a, b) and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right]$$

for some ξ in (a, b) and $h = \dfrac{b-a}{n}$, with the error term is $\left|\dfrac{(b-a)h^2 f''(\xi)}{12}\right|$.

10. Simpson's rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{h^5}{90} f^{(iv)}(\xi)$$

for some ξ in (a, b) and $h = \dfrac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right]$$

for some ξ in (a, b) and $h = \dfrac{b-a}{n}$, with the error term $\left|\dfrac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$.

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,\ldots$$

Continued ...

13. Euler's method

$$y_{i+1} = y_i + hf\left(x_i, y_i\right)$$

with local error $\frac{h^2}{2}Y''\left(\xi_i\right)$ for some ξ_i in $\left(x_i, x_{i+1}\right)$.

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf\left(x_i, y_i\right),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf\left(x_{i+1}, y_i + k_3\right),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

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